**Al-Farabi KAZAKH NATIONAL UNIVERSITY**

**Faculty of mechanics and mathematics**

**Educational program for the specialty «5В060100-Mathematics»**

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|  | Approved by the Faculty Scientific Council meeting Protocol №\_\_\_ from \_\_\_\_\_\_\_\_\_\_\_\_ 2013**Dean of the faculty** **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_A. Kadyrbekuly** |

**SULLABUS**

**includes courses**

**«\_\_\_\_\_\_\_\_\_» «Calculus of variations and optimization methods»** (3 credits)

3 course, k/s, r/s, autumn semester

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**PASSPORT of the course:**

**Aim** (in concordance with the aim of the module).

The analysis of the classical variational problems and optimization control problems.

**Problems:**

The consideration of the methods of the classical variational problems and optimization control problems. The application of these methods for solving practical variational problems and optimization control problems.

**Results:**

Methods of solving of variational problems and optimization control problems.

**Pre Essential Elements**:

Differential equations. Mathematical analysis. Mechanics.

**Post** **Essential Elements**:

Numerical methods for extremum problems. Inverse problems. Optimization control problems for distributed systems.

**STRUCTURE, VOLUME AND CONTENT OF THE COURSE**

|  |  |
| --- | --- |
| **week** | **Course «3» - « Calculus of variations and optimization methods»,** 3 credits |
| **subject** | **hours** | **tasks** |
| **Part I. Introduction** |
| **1** | **Lecture 1. Practical examples of the extremum problems**. Maximization of the flight of the body. Brachistochrone problem. Maximization of the flight of the missile.Practical work 1. Practical examples of the extremum problems. | 21 |  |
| **2** | **Lecture 2. Minimization of functions**. Stationary condition. Examples. Maximization of the flight of the body. Minimization of the function of many variables.Practical work 2. Minimization of functions and stationary condition. | 21 | Use stationary condition for the concrete function*.* Check the properties of the stationary points. Chose the function with given property.  |
| **Part II. Calculus of variations** |
| **3** | **Lecture 3. Euler equation for Lagrange problem.** Lagrange problem. Euler equation. Examples. The fall of the body. Fermat principle and the refraction of light low.Practical work 3. Euler equation for Lagrange problem. | 21 | Determine Euler equation for the concrete Lagrange problem. Find the general solution of Euler equation, which depends from two constants. Find these constants by means of the given boundary conditions. Find the corresponding solution of the boundary problem. Calculate the corresponding value of the given functional. Calculate the value of the given functional for the linear function which satisfies the given boundary conditions. Compare these results. |
| **4** | **Lecture 4. Lagrange problem for the functions family.** Problem statement. The system of Euler equations. Example. Principle of the least action. Practical work 4. Lagrange problem for the functions family. | 21 | Determine the system of Euler equations for the concrete problem. Find general solution of this system. Find the solution of Euler equations, which satisfies boundary conditions. Show the graphs of these solutions. Calculate the corresponding value of the given integral. |
| **5** | **Lecture 5. Lagrange problem with high derivatives.** Problem statement. Euler – Poisson Equation. Example. Bending of the elastic beam.Practical work 5. Lagrange problem with high derivatives. | 21 | Determine the system of Euler – Poisson equation for the concrete problem. Find general solution of this equation. Find the solution of Euler – Poisson equation, which satisfies given boundary conditions. Show the graph of this solution. Calculate the corresponding value of the given integral. |
| **6** | **Lecture 6. Lagrange Problem for functions with many variables.** Problem statement. Ostrogradsky equation. Dirichlet integral. The oscillation of the string.Practical work 6. Lagrange Problem for functions with many variables. | 21 | Determine Ostrogradsky equation for the concrete problem. |
| **7** | **Lecture 7. Bolza Problem.** Problem statement. Necessary conditions of extremum. Transversality conditions. Example. River crossing problem.Practical work 7. Bolza Problem. | 21 | Determine Euler equation and the transversality conditions for the concrete problem. Find the general solution of this equation. Find the solution of boundary problem. Calculate the corresponding value of the given integral. |
| **8** | **Lecture 8. Variational problems with isoperimetric conditions.** Problems with isoperimetric condition. Lagrange multipliers method. A spectrum problem. The problem with many isoperimetric conditions.Practical work 8. Variational problems with isoperimetric conditions. | 21 | Determine Euler equation for the concrete problem. Verify the sign of the Lagrange multiplier with using multiplication of Euler equation by unknown function and integration. Find the general solution of Euler equation; it depends from two constants and Lagrange multiplier. Using given boundary conditions and isoperimetric condition find three unknown constants. Find the set of the solutions of the conditions of the extremum. Calculate the value of the given integral for all solution of the conditions of the extremum. |
| **9** | **Lecture 9. Variational problems with pointwise constraints.** Problem statement. Lagrange multipliers method. Example. Oscillation of the pendulum.Practical work 9. Variational problems with pointwise constraints.  | 21 | Denote the system of the extremum conditions (concrete Euler equations with boundary and addition conditions). Multiply the first Euler equation by the given parameter a, and second equation by b. Add these equalities with using of the condition (\*). Find Lagrange multiplier λ. Put λ to Euler equations.Find the general solutions of two Euler equations. It equals the sum of the general solution of the corresponding homogeneous equation and the constant, which satisfies the given equation. Find four constants from general solutions of Euler equations with using of the boundary conditions. Put these constants to the formulas of the general solutions. It will be the solution of the problem. |
| **Part II. Optimization control problems** |
| **10** | **Lecture 10. Easiest optimization control problems.** Maximization of the flight of the missile (problem statement). Pontyagin’s maximum principle. Example. Iterative method for solving the optimality conditions.Practical work10**.** Easiest optimization control problems. | 21 | Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. |
| **11** | **Lecture 11. Optimization control problems for the vector case.** Problem statement. Pontyagin’s maximum principle. Example. Maximization of the flight of the missile (solving).Practical work 11.Optimization control problems for the vector case.  | 21 | Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. |
| **12** | **Lecture 12. Optimization control problem with fixed final state.** Problem Statement.Maximum principle. Example. Time optimization problem. Firing method.Practical work 12.Optimization control problem with fixed final state. | 21 | Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality with using of firing method. |
| **13** | **Lecture 13. Differentiation of functionals and abstract optimization problems.** Gradient methods for functions. Gateau derivatives of functionals. Examples. Gradient methods for functionals.**Practical work 13.** Differentiation of functionals and abstract optimization problems.  | 21 | Calculate Gateau derivative for the concrete functional. Determine gradient method and projection gradient method. |
| **14** | **Lecture 14**. **Variational inequalities**. Variational inequalities and constraints minimization of functional. Examples. Variational inequalities and constraints minimization of functional.Practical work 14.Variational inequalities. | 21 | Calculate Gateau derivative for the concrete functional. Determine variational inequality. Find the solution of variational inequality. Calculate the value of minimizing functional. |
| **15** | **Lecture 15. Existence and uniqueness of extremum problems.** Existence theorem for abstract optimization problems. Uniqueness theorem for abstract optimization problems. Example.Practical work 15. Existence and uniqueness of extremum problems. | 21 | Prove the convexity and continuity of the concrete functional. Prove the convexity, closeness and boundedness of the concrete set of admissible control. Prove the existence of the optimal control. |

**Key words**

Calculus of variations. Optimization control problem. Necessary condition of extremum. Euler equation. Maximum principle. Gateaux derivative. Gradient method.

**Literature**

**Basic**

1. Алексеев В. М., Тихомиров В. М., Фомин С. В. Оптимальное управление. – М., Наука, 1979.
2. Будылин А.М. Вариационное исчисление. – Санкт-Петербург, СПбГУ, 2001. – [http://www.newlibrary.ru/book/budylin\_a\_m\_/variacionnoe\_ischislenie.html](http://www.newlibrary.ru/book/budylin_a_m_/variacionnoe_ischislenie.html%20) .
3. Васильев Ф.П. Методы оптимизации. В двух томах. – М.: МЦНМО, 2011.
4. Лутманов С.В. Курс лекций по методам оптимизации. – Ижевск, 2001.
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6. Kirk D. E. Optimal Control Theory: An Introduction. – New Jersey, Englewood Cliffs, 2004. <http://www.amazon.com/Optimal-Control-Theory-Introduction-Engineering/dp/0486434842>
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**Additional**

1. Габасов Р., Кириллова Ф. Качественная теория оптимальных процессов. – М., Наука, 1907. – 507 с.
2. Иоффе А. Д., Тихомиров В. М. Теория экстремальных задач. – М., Наука, 1974. – 480 с.
3. Канторович Л. В., Акилов Г. П. Функциональный анализ. – М., Наука, 1977. – 744 с.
4. Ахиезер Н.И. Лекции по вариационному исчислению. – М., ГИТТЛ, 1956.
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6. Краснов М.Л., Макаренко Г.И, Киселев А.И. Вариационное исчисление. – М., Наука, 1973.
7. Лаврентьев М.А., Люстерник Л.А. Основы вариационного исчисления. Том 2. – М., ОНТИ, 1935.
8. Miersemann E. Calculus of Variations. Lecture Notes. – Leipzig, 2012.

**Tasks and methodical guidelines for individual student work**

**Task 2. Minimization of functions**

|  |  |  |
| --- | --- | --- |
| **variant** | **Question 1** | **Question 2** |
|  | Use stationary conditionfor the concrete function ***f****.*Check the properties of the stationary points. | Chose the function withgiven property. May be this result is impossible. |
| 1 |  | The stationary conditionhas a unique solution, which is not a point of the minimum. |
| 2 |  | The stationary conditiondoes not have any solutions. |
| 3 |  | The stationary conditionhas two solutions, one of them it is the point of the minimum. |
| 4 |  | The stationary conditionis the necessary andsufficient of the minimum. |
| 5 |  | The stationary conditionhas an infinite set of solutions. |
| 6 |  | The stationary conditiondoes not have any solutions butthe minimum of the function exists. |
| 7 |  | The stationary condition has three solutions: local minimum, local maximum and absolute minimum. |
| 8 |  | The stationary condition has two solutions, it is the sufficient condition of minimum. |
| 9 |  | The stationary condition for the function with two points of the absolute minimum. |
| 10 |  | The stationary condition has three solutions: local minimum, local maximum and absolute maximum. |
| 11 |  | The stationary conditionhas two solutions, one of them it is the point of the maximum. |
| 12 |  | The stationary conditiondoes not have any solutions, butthe maximum of the function exists. |
| 13 |  | The stationary condition is not applicable, but the maximum of the function exists. |
| 14 |  | The stationary condition has two solutions, one of them is not the point of the minimum. |
| 15 |  | The stationary condition is not the sufficient condition of the maximum. |
| 16 |  | The stationary condition for a function with two point of the maximum. |

**Task 2. Euler equation for Lagrange problem**

Find the function  which minimizes the functional



And satisfies the boundary conditions



The values of the parameters.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **variant** |  |  |  |  |  | **name** |
| 1 |  | 0 |  | 0 | 1 |  |
| 2 |  | 0 | 1 | 0 | 1 |  |
| 3 |  | 0 |  | 0 | -1 |  |
| 4 |  | 0 | 1 | 0 | 1 |  |
| 5 |  | 0 |  | 0 | 1 |  |
| 6 |  | 0 | 1 | 0 | 1 |  |
| 7 |  | 0 | 1 | 0 | 1 |  |
| 8 |  | 0 |  | 1 | 0 |  |
| 9 |  | 0 | 1 | 0 | 1 |  |
| 10 |  | 0 | *π* | 0 | 1 |  |
| 11 |  | 0 |  | 0 | 1 |  |
| 12 |  | 0 |  | 0 | 1 |  |
| 13 |  | 0 |  | 0 | -1 |  |
| 14 |  | 0 |  | 0 | -1 |  |
| 15 |  | 0 |  | 0 | 1 |  |
| 16 |  | 0 |  | 0 | 1 |  |
| 17 |  | 0 |  | 1 | 0 |  |
| 18 |  | 0 |  | 1 | 0 |  |
| 19 |  | 0 |  | 0 | -1 |  |
| 20 |  | 0 |  | 0 | -1 |  |

It is necessary to make the following actions:

1. Determine Euler equation.
2. Find the general solution of Euler equation, which depends from two constants.
3. Find these constants by means of the given boundary conditions.
4. Find the corresponding solution of the boundary problem.
5. Calculate the corresponding value of the given functional.
6. Calculate the value of the given functional for the linear function which satisfies the given boundary conditions.
7. Compare these results.

It will be recalled that the general solution of the differential equation  is

.

The general solution of the differential equation  is



### Task 3. Lagrange problem for the functions family

Find the functions   which minimize the integral



with boundary conditions



The values of the parameters

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| variant |  |  |  |  |  |  |  |
| 1 |  | 0 | 1 | -1 | 0 | -1 | 0 |
| 2 |  | 0 | 1 | -1 | 0 | -1 | 0 |
| 3 |  | -1 | 0 | -1 | 0 | -1 | 0 |
| 4 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 5 |  | 0 | 2 | 0 | 2 | 0 | 1 |
| 6 |  | 0 | 1 | 0 | 1 | 1 | 0 |
| 7 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 8 |  | 0 | 1 | 1 | 0 | 1 | 0 |
| 9 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 10 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 11 |  | -1 | 0 | 0 | 1 | 0 | 1 |
| 12 |  | 0 | 1 | -1 | 0 | -1 | 0 |
| 13 |  | 0 | 2 | 0 | 1 | 0 | 1 |
| 14 |  | 0 | 2 | 0 | 1 | 0 | 1 |
| 15 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 16 |  | 0 | 1 | 0 | 1 | 1 | 0 |
| 17 |  | 0 | 2 | 0 | 1 | 0 | 1 |
| 18 |  | 0 | 1 | 0 | 2 | 0 | 1 |

Steps of the task.

1. Determine the system of Euler equations.
2. Find general solution of this system.
3. Find the solution of Euler equations, which satisfies boundary conditions.
4. Show the graphs of these solutions.
5. Calculate the corresponding value of the given integral.

### Task 4. Minimization of the functional which depends of the second derivative of the unknown function

Find the function  which minimize the integral



with boundary conditions



The values of parameters.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| variant |  |  |  |  |  |  |  |
| 1 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 2 |  | 0 | 1 | -1 | 0 | -1 | 0 |
| 3 |  | -1 | 0 | -1 | 0 | -1 | 0 |
| 4 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 5 |  | 0 | 2 | 0 | -1 | 0 | 1 |
| 6 |  | -π | 0 | 0 | 1 | 1 | 0 |
| 7 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 8 |  | -1 | 1 | 1 | 0 | 1 | 0 |
| 9 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 10 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 11 |  | 0 | 1 | -1 | 0 | -1 | 0 |
| 12 |  | -1 | 0 | -1 | 0 | -1 | 0 |
| 13 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 14 |  | 0 | 1 | 0 | -1 | 0 | 1 |
| 15 |  | 0 | π | 0 | 1 | 1 | 0 |
| 16 |  | 0 | 1 | 0 | 1 | 0 | 1 |
| 17 |  | -1 | 1 | 1 | 0 | 1 | 0 |
| 18 |  | 0 | 1 | 0 | 1 | 0 | 1 |

Steps of the task.

1. Determine the system of Euler – Poisson equation.
2. Find general solution of this equation.
3. Find the solution of Euler – Poisson equation, which satisfies given boundary conditions.
4. Show the graph of this solution.

Calculate the corresponding value of the given integral.

### Task 5. Bolza Problem.

**Variants 1-7**. Find the function, which satisfies the boundary condition  and minimize the integral

.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| variant |  |  |  |  |  |
| 1 |  | *v* | 0 | 1 | 0 |
| 2 |  | -*v* | -1 | 0 | 1 |
| 3 |  | -2*v* | 0 | 1 | 0 |
| 4 |  | 2*v* | -1 | 0 | 1 |
| 5 |  | -3*v* | 1 | 0 | 0 |
| 6 |  | 3*v* | 0 | 1 | 1 |
| 7 |  | -*v* | 0 | 1 | 0 |

**Variants 9-14**. Find the function, which satisfies the boundary condition  and minimize the integral

.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| variant |  |  |  |  |  |
| 8 |  | *v* | 0 | 1 | 0 |
| 9 |  | -*v* | -1 | 0 | 1 |
| 10 |  | -2*v* | 0 | 1 | 0 |
| 11 |  | 2*v* | -1 | 0 | 1 |
| 12 |  | -3*v* | 1 | 0 | 0 |
| 13 |  | 3*v* | 0 | 1 | 1 |
| 14 |  | -*v* | 0 | 1 | 0 |

**Variants 15-21**. Find the function  which minimize the integral

###

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variant |  |  |  |  |
| 15 |  |  | 0 | 1 |
| 16 |  |  | -1 | 0 |
| 17 |  |  | 0 | 1 |
| 18 |  |  | 0 | 1 |
| 19 |  |  | 1 | 0 |
| 20 |  |  | 0 | 1 |
| 21 |  |  | 0 | 1 |

Steps of the task.

1. Determine Euler equation, and find its general solution.
2. Find the solution of this equation, which satisfies the following boundary conditions: two transversality conditions for the variants 15-21 or one transversality condition and the given boundary condition for other variants.
3. Show the graph of this solution.
4. Calculate the corresponding value of the given integral.
5. Calculate the value of the integral for an arbitrary linear function for the variants 15-21 and linear function, which satisfies given boundary condition for other variants.

**Task 6. Variational problems with isoperimetric conditions**

Consider the problem of the minimization of the integral



with boundary conditions

  (\*)

or

  (\*\*)

and isoperimetric condition



Table. 8.1. The values of the parameters.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variant | boundary conditions | *a* | *b* | *M* |
| 1 | \* | 0 | 2π | 2 |
| 2 | \*\* | 0 | π | 1 |
| 3 | \* | π | 2π | 1 |
| 4 | \*\* | 0 | π | 2 |
| 5 | \* | 0 | π | 2 |
| 6 | \*\* | 0 | π | 3 |
| 7 | \* | 0 | π | 3 |
| 8 | \*\* | 0 | 2π | 1 |
| 9 | \* | 0 | 2π | 3 |
| 10 | \*\* | -π | π | 1 |
| 11 | \* | -π | 0 | 1 |
| 12 | \*\* | -π | π | 2 |
| 13 | \* | -π | 0 | 2 |
| 14 | \*\* | 0 | 2π | 1 |
| 15 | \* | -π | 0 | 3 |
| 16 | \*\* | 0 | 2π | 2 |
| 17 | \* | -π | π | 1 |
| 18 | \*\* | 0 | 2π | 3 |
| 19 | \* | -π | π | 2 |
| 20 | \*\* | 0 | π | 4 |

Steps of the task.

1. Give the concrete problem statement.
2. Write Euler equation.
3. Verify the sign of the Lagrange multiplier with using multiplication of Euler equation by unknown function and integration.
4. Find the general solution of Euler equation; it depends from two constants and Lagrange multiplier.
5. Using given boundary conditions and isoperimetric condition find three unknown constants.
6. Find the set of the solutions of the conditions of the extremum.
7. Calculate the value of the given integral for all solution of the conditions of the extremum.
8. Chose the minimum of these values; find the solution of the problem.

**Task 7. Variational problems with pointwise constraints**

Find the functions   which minimize the integral



on the set of the functions with boundary conditions



and additional condition

 . (\*)

Values of the parameters

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| variant | *Т* | *X* | *Y* | *α* | *a*  | *b* | *c* |
| 1 | 1 | 1 | 1 | -π2 | π | π | 1 |
| 2 | π | 1 | 2 | -1 | 1 | 1 | 1 |
| 3 | π/2 | 1 | 1 | -2π2 | π | π | 2 |
| 4 | π/2 | 2 | 2 | -2 | 2 | 2 | 2 |
| 5 | 4 | 2 | 2 | -4π2 | π | π | 2 |
| 6 | π | 1 | 1 | -1 | 2 | 2 | 2 |
| 7 | 2 | 1 | 1 | -π2 | π/2 | π | 2 |
| 8 | π | 2 | 2 | -1 | 1 | 1 | 2 |
| 9 | 4 | π | π | -π2 | π | π/2 | 2 |
| 10 | 1 | 1 | 1 | -1 | 1 | 1 | 1 |
| 11 | π | 1 | 1 | -π2 | π | π | 1 |
| 12 | 2 | 1 | 1 | -1 | 1 | 1 | 1 |
| 13 | π | 1 | 2 | -π2 | π | π | 1 |
| 14 | π | 2 | 1 | -1 | 1 | 1 | 1 |
| 15 | π | 2 | 2 | -2 | 2 | 2 | 2 |
| 16 | 2 | 2 | 2 | -π2 | π/2 | π | 2 |
| 17 | π | 2 | 1 | -1 | π | π | 1 |
| 18 | π | 1 | 2 | -π2 | π | π | 2 |

Steps of the task

1. Denote the concrete problem statement.
2. Denote the system of the extremum conditions (concrete Euler equations with boundary and addition conditions).
3. Multiply the first Euler equation by the given parameter *a*, and second equation by *b*. Add these equalities with using of the condition (\*). Find the value *λ*.
4. Put *λ* to Euler equations.
5. Find the general solutions of two Euler equations. It equals the sum of the general solution of the corresponding homogeneous equation and the constant, which satisfies the given equation.
6. Find four constants from general solutions of Euler equations with using of the boundary conditions.
7. Put these constants to the formulas of the general solutions. It will be the solution of the problem.

### Task 8. Optimization control problems

Consider the control system described by the differential equations



with initial conditions



The set of the admissible control is described by the inequalities



We have the problem of the minimization of the value

.

Table of the parameters.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variants |  |  |  |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |
| 11 |  |  |  |  |
| 12 |  |  |  | - |
| 13 |  |  |  |  |
| 14 |  |  |  |  |
| 15 |  |  |  |  |
| 16 |  |  |  |  |
| 17 |  |  |  |  |
| 18 |  |  |  |  |

Steps of the task.

1. Write the concrete problem statement.
2. Determine the function *Н.*
3. Determine the adjoint system.
4. Determine the maximum principle.
5. Find the control from the maximum principle.
6. Determine the iterative method for solving the conditions of the optimality.

### Task 9. Optimization control problems with fixed final state

Consider the problem from the Task 8 with additional final state conditions: the values of the state functions equal to zero for the final time. Determine necessary conditions of optimality.Determine the iterative method with using of firing method.

### Task 10. Gateaux derivatives and gradient methods

Find Gateaux derivatives of the functional *I* in the space *V*.

1. 
2. 
3. 

Determine the gradient method. and projection

### Task 11. Projection gradient method

Consider the problem from the Task 10 with additional condition: the unknown function belongs to the unit interval. Determine the projection gradient method.

**Form of control and competence**

Tasks for individual student works.

Control – 2 times.

Exam during the session.

**Criterion for the estimate of the knowledge**

Individual student works – 60 %

Examination – 40 %

**Consultations are taken during the office-hours of the lecturer and by mail.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| № | Вид контроля | Максимальный балл | Минимальный балл  | Примечание |
| 1 | Рубежный контроль 1 | 100 | 50 | Сумма оценок по все видам заданий за 1 - 7 недели |
| 2 | Рубежный контроль 2 | 100 | 50 | Сумма оценок по все видам заданий за 8 - 15 недели |
| 3 | Оценка текущей успеваемости | (РК1+РК2)/2=100 | 50 | Среднее арифметическое РК1 и РК2 |
| 4 | Оценка итогового контроля (экзаменационная оценка) | 100 | 50 |  |
| 5 | Итоговая оценка по дисциплине | 100 | 50 | Среднее арифметическое оценки текущей успеваемости и экзаменационной оценки |

**Шкала оценки знаний:**

|  |  |  |  |
| --- | --- | --- | --- |
| Оценка по буквенной системе | Цифровой эквивалент баллов | %-ное содержание | Оценка по традиционной системе |
| А | 4,0 | 95-100 | Отлично  |
| А- | 3,67 | 90-94 |
| В+ | 3,33 | 85-89 | Хорошо |
| В | 3,0 | 80-84 |
| В- | 2,67 | 75-79 |
| С+ | 2,33 | 70-74 | Удовлетворительно |
| С | 2,0 | 65-69 |
| С- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D- | 1,0 | 50-54 |
| F | 0 | 0-49 | Неудовлетворительно |
| I (Incomplete) | - | - | «Дисциплина не завершена»(*не учитывается при вычислении GPA)* |
| P (Pass) | **-** | **-** | «Зачтено»(*не учитывается при вычислении GPA)* |
| NP (No Рass) | **-** | **-** | «Не зачтено»(*не учитывается при вычислении GPA)*  |
| W (Withdrawal) | - | - | «Отказ от дисциплины»(*не учитывается при вычислении GPA)* |
| AW (Academic Withdrawal) |  |  | Снятие с дисциплины по академическим причинам(*не учитывается при вычислении GPA)* |
| AU (Audit) | - | - | «Дисциплина прослушана»(*не учитывается при вычислении GPA)* |
| Атт.  |  | 30-6050-100 | Аттестован |
| Не атт. |  | 0-290-49 | Не аттестован |
| R (Retake) | - | - | Повторное изучение дисциплины |

Session № \_\_ of « \_\_ » \_\_\_\_\_\_\_\_\_\_\_ 2013.

**Head of the Department S. Muhambetzhanov**

**Lecturer S. Serovajsky**